Sequentially split $*$-homomorphisms (Part I)

Workshop on Structure and Classification of C^*-algebras

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WWU Münster

April 2015

A word of warning: This talk describes work in progress, and the proofs of the results still need to be checked in detail. Do not quote them yet!

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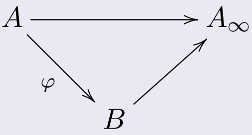

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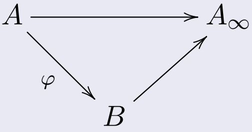

of $*$-homomorphisms.

Remark

If one restricts to separable C^*-algebras, one gets an equivalent definition upon replacing A_∞ by A_ω, for any free filter ω on $\mathbb{N}$.

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Theorem (Toms-Winter)

Let A be a separable C^*-algebra and let $\mathcal{D}$ be a strongly self-absorbing C^*-algebra. Then A is $\mathcal{D}$-stable if and only if the first factor embedding $\mathrm{id}_A \otimes \mathbf{1}_{\mathcal{D}} : A \to A \otimes \mathcal{D}$ is sequentially split.

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We will see more examples later.

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Let $\{A_n, \kappa_n\}$ and $\{B_n, \theta_n\}$ be two inductive systems of separable C^*-algebras. Let $\varphi_n : A_n \to B_n$ be a sequence of $*$-homomorphisms compatible with the connecting maps, and denote by $\varphi : \varinjlim A_n \to \varinjlim B_n$ the induced map on the limit C^*-algebras. If every φ_n is sequentially split, then so is φ.

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(I) *For each ideal J of A, the restriction $\varphi|_J : J \to \overline{B\varphi(J)B}$ and the induced map $\varphi_{\mathrm{mod}\,J} : A/J \to B/\overline{B\varphi(J)B}$ are sequentially split.*

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(VI) *The induced map between the simplices of tracial states $T(\varphi) : T(B) \to T(A)$ given by $\tau \mapsto \tau \circ \varphi$ is surjective.*

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(16) *stability under tensoring with the compacts* $\mathcal{K}$*.*

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Remark

Out of all the permanence properties, the above is the furthest from being trivial. A technique by Kirchberg makes it possible to reduce this to the case of A and B being Kirchberg algebras. From then on, one needs to use Kirchberg-Phillips classification paired with (weak) semiprojectivity arguments.

Definition (Watatani)

Let B be a unital C^*-algebra and $A \subset B$ a unital sub-C^*-algebra. Let $E : B \to A$ be a conditional expectation. Then E is said to have a quasi-basis, if there exist elements $u_1, v_1, \ldots, u_n, v_n \in B$ such that

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If $A \hookrightarrow B$ is some inclusion of unital C^*-algebras such that there exists a conditional expectation $E : B \to A$ with a quasi-basis, one also says that this inclusion has finite Watatani Index.

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If $A \hookrightarrow B$ is an inclusion of unital C^*-algebras with finite Watatani-Index, then there is a unique conditional expectation $E : B \to A$. Moreover, its index $\mathrm{ind}(E)$ is a positive, invertible, central element in B.

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Definition (Osaka-Kodaka-Teruya)

Let B be a unital C^*-algebra and $A \subset B$ a unital sub-C^*-algebra. Let $E : B \to A$ be a conditional expectation and assume that the inclusion $A \hookrightarrow B$ has finite Watatani Index. This inclusion is said to have the Rokhlin property, if there exists a projection $p \in B_\infty \cap B'$ such that $E_\infty(p) = \mathrm{ind}(E)^{-1}$.

Example (Osaka-Kodaka-Teruya)

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As it turns out, this result fits nicely into the setting of sequentially split $*$-homomorphisms.

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Paired with the permanence results of this talk, this observation recovers and extends the permanence results proved by Osaka, Kodaka, Teruya.

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$\rightsquigarrow$ More examples like this to come in the next talk.

Thank you for your attention!